External Flows

Thursday, February 12, 2026 12:01 PM

Review

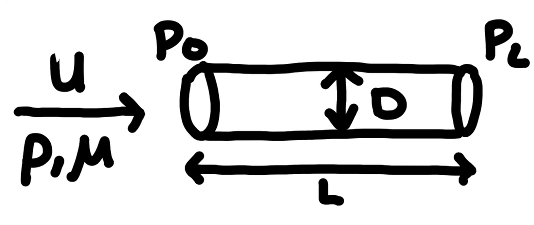
Circle of Flow Calculations

· forces, stresses, pressures $\Longleftrightarrow$ velocity, volumetric/mass flowrates (size)

$\Updownarrow$ inertial stress   properties U $\Updownarrow$ Q

f (pipes) C_D (spheres) C_f (plate) $\overset{\text{dimensionless}}{\Longleftrightarrow}$ $Re = \frac{\rho U L}{\mu}$

· internal flows



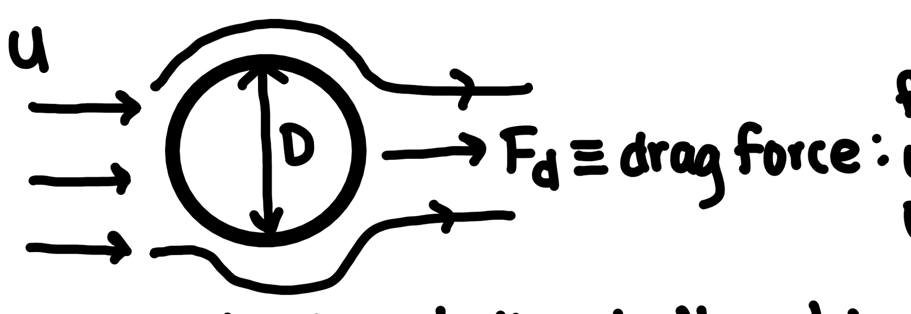
difference in pressure drives the flow

$f \equiv$ friction factor

$f(Re) \rightarrow$ ratio of friction stress to inertial

▫ unidirectional and confined

· external flow



$F_d \equiv$ drag force : fluid dynamic force on object in direction of fluid flow

▫ U is velocity relative to the object

$C_D \equiv$ drag coefficient

$C_D = \text{fxn}(Re)$; dimensionless

Drag Force

· 2 effects:

1. "form drag" : pressure difference
2. "friction drag" : viscous shear stress
   ↳ result of sliding over the surface of the object

· Drag Coefficient, C_D

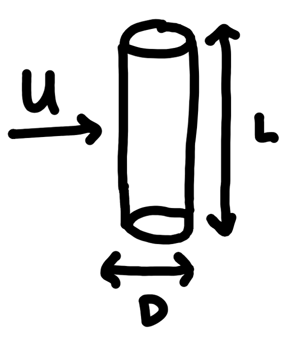
$\text{stress} = \frac{F_D}{A_\perp}$; for a sphere : $A_\perp = \frac{\pi D^2}{4}$

$C_D = \frac{F_D / A_\perp}{\frac{1}{2}\rho U^2}$; for a sphere : $\frac{8}{\pi}\frac{F_D}{\rho U^2 D^2}$

· C_D'n Different Regimes - Sphere

1. $Re < 1$, $C_D = 24/Re$, Stokes regime, viscous stresses dominate
   $F_D = 3\pi\mu U D$
2. $0.1 < Re < 4 \cdot 10^3$, $C_D = 0.28 + 6/\sqrt{Re} + 21/\sqrt{Re}$, viscous + inertial
3. $750 < Re < 10^5$, Newton's Regime, $C_D \cong 0.445$, inertial
4. $2 \cdot 10^5 < Re < 4 \cdot 10^5$, Eiffel's, C_D falls
5. $Re > 10^6$, $C_D = 0.19 - \frac{8 \cdot 10^4}{Re}$

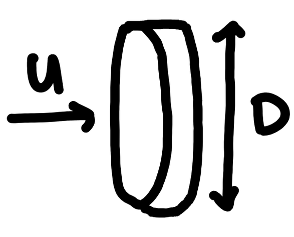
· Long Cylinder ($L \gg D$)



$C_D = \frac{F_D / A_\perp}{\frac{1}{2}\rho U^2} = 2\frac{F_D / L}{\rho U^2 D}$

$A_\perp = LD$

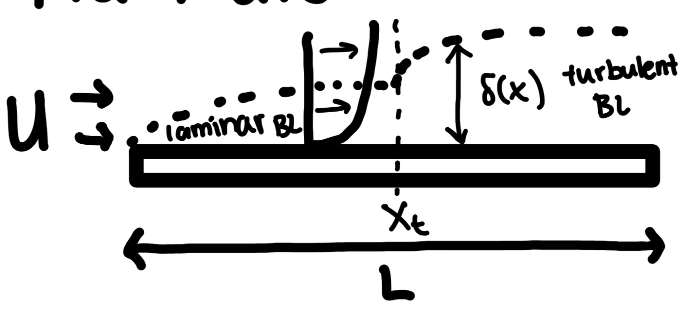
· Disk (thickness neglible)



$C_D = \frac{F_D / A_\perp}{\frac{1}{2}\rho U^2} = \frac{8}{\pi}\frac{F_D}{\rho U^2 D^2}$

$A_\perp = \pi D^2/4$

· Flat Plate



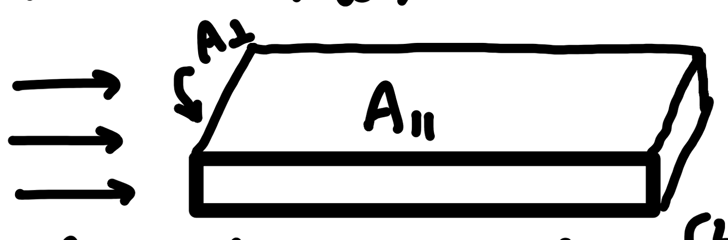
BL = boundary layer

$\delta(x)$ = BL thickness

shear rate $= \underbrace{U/\delta(x)}_{\text{very large}} \rightarrow$ velocity gradient

$\langle\tau_w\rangle$ = average shear stress

$\langle\tau_w\rangle = F_D / A_w$; A_w is wetted area



$A_\parallel \gg A_\perp \Rightarrow A_w \approx A_\parallel = \begin{cases} LW & \text{1-side} \\ 2LW & \text{2-side} \end{cases}$

frictional drag >> form drag

$C_f = \frac{\langle\tau_w\rangle}{\frac{1}{2}\rho U^2} = \frac{2F_D}{\rho U^2 A_w}$

$C_f = \text{fxn}(Re)$

▫ $C_f = 1.328\, Re^{-1/2}$ $(L < x_t)$

↳ if all laminar and $Re > 100$

▫ $C_f = \frac{0.455}{(\log Re)^{2.58}} - \frac{B}{Re}$ $(L > x_t)$

↳ if turbulent and laminar

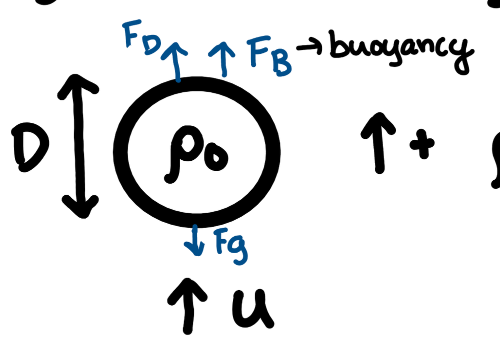
▫ $B = 1050$ for $Re_t = 3 \cdot 10^5$

▫ $B = 8700$ for $Re_t = 3 \cdot 10^6$

Terminal Velocity

· steady state velocity balancing forces of motion and drag force

e.g. sphere falling



$\uparrow +$ ρ = fluid density

$0 \overset{\text{s.s.}}{=} \sum_i F_i$

$0 = F_G + F_B + F_D$

$0 = -\rho_0\left(\pi D^3/6\right)g + \rho\left(\pi D^3/6\right)g + \frac{\pi}{8}\rho U^2 D^2 C_D$

▫ rearrange, solve for U^2

$U^2 = \frac{4}{3}\left(\frac{\rho_0}{\rho} - 1\right)\frac{gD}{C_D}$

▫ C_D is a fxn of Re, so fxn of U! issue!

↳ don't know what regime either,
so don't know relationship (linear, reciprocal, etc.)

$\left[C_D\left(\frac{\rho D}{\mu}\right)^2\right] U^2 = \frac{4}{3}\left(\frac{\rho_0}{\rho} - 1\right)\frac{gD}{C_D}\left[C_D\left(\frac{\rho D}{\mu}\right)^2\right]$

$C_D Re^2 = \frac{4}{3}\underbrace{\left(\frac{\rho_0}{\rho} - 1\right) g D^3 \left(\frac{\rho}{\mu}\right)^2}_{Ar \,\equiv\, \text{Archimedes \#}} \rightarrow$ dimensionless!

$\boxed{C_D Re^2 = \frac{4}{3} Ar}$

1. $0 < Ar < 20$, $Re = Ar/18$, Stokes
2. $20 < Ar < 2 \cdot 10^5$, iterative approach
3. $2 \cdot 10^5 < Ar < 10^{10}$, $Re = (3Ar)^{1/2}$, Newton's ($C_D \approx$ const)