Pipe Flows

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Control Volume

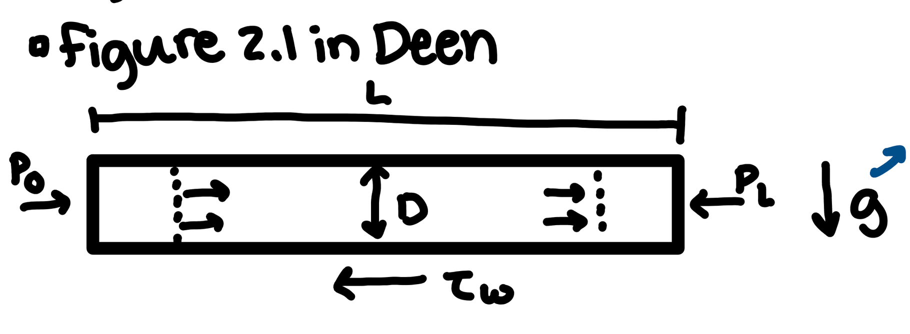
· Review

$$\tau_w = \frac{F}{A} = \frac{\mu U}{H} = \mu\dot{\gamma}$$

$$Re = \frac{\tau_{inertial}}{\tau_{viscous}} = \frac{\rho U^2}{\mu U/H} = \frac{\rho U H}{\mu}$$

· Cylindrical Pipe Flow

▫ Figure 2.1 in Deen



0 when orthogonal to direction of flow

▫ cylinder, solid walls, steady flow, fully developed, $L \gg D$, $Q = U \times A$, constant ρ

↳ volumetric flow rate

▫ pressure is the driving force

Friction Factor

Force Balance in x-direction on CV:

$$(P_0 - P_L)\underbrace{\frac{\pi D^2}{4}}_{\text{cross sectional area}} - \tau_w \underbrace{\pi D}_{\text{perimeter of pipe}} L = \sum F_x \overset{S.S.}{=} 0$$

$$\tau_w = \frac{-D}{4}\frac{\Delta P}{L}\ ;\ \Delta P = P_L - P_0 < 0$$

$$\boxed{\tau_w = \frac{D}{4}\frac{|\Delta P|}{L}}$$ → horizontal pipe, no gravity

▫ dimensional analysis

$$\tau_w = fxn(u, D, \mu, \rho)$$

$\tau_w \to \frac{M}{LT^2}$, $u \to \frac{L}{T}$, $D \to L$, $\mu \to \frac{M}{LT}$, $\rho \to \frac{M}{L^3}$

inertial stress $\sim \rho u^2$

$G = 5 - 3 = 2$

$$N_1 = \frac{2\tau_w}{\rho u^2}$$

· 2 is convential

⟶ this is friction factor f!

$$N_1 = \frac{2\tau_w}{\rho u^2} = fxn(N_2)$$

$$N_2 = Re = \frac{Du\rho}{\mu}$$

$$\frac{2\tau_w}{\rho u^2} = fxn(Re) = f$$

· Laminar Flow (not turbulent)

▫ Hagen-Poiseuille (1840)

$$f = \frac{16}{Re}$$

$$\frac{2\tau_w}{\cancel{\rho} u^{\cancel{2}}} = \frac{16\mu}{D\cancel{u}\cancel{\rho}}$$

$$\tau_w = \frac{8\mu U}{D}\ \Big\}\ \text{viscous stress}$$

$$\frac{-D\Delta P}{4L} = \frac{8\mu U}{D}$$

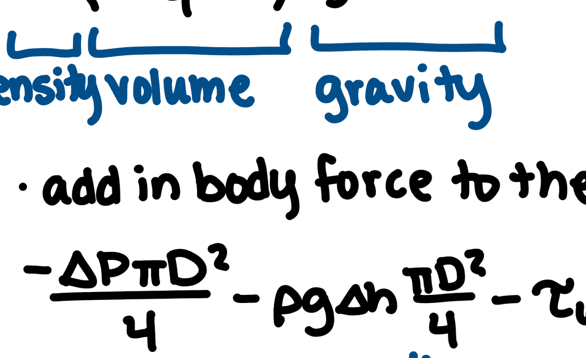
$$\boxed{U = \frac{D^2}{32\mu}\frac{|\Delta P|}{L}}$$

$$Q = UA = \frac{\pi}{128}\frac{D^4}{\mu}\frac{|\Delta P|}{L}$$

▫ analogous to current resistance

$$I = \frac{\text{potential}}{\text{resistance}}$$

Non-horizontal Pipe



· how much of g obey pipe flow direction?

$\Delta h = h_L - h_0$

$\sin\alpha = -\Delta h/L$

$$\underbrace{\rho}_{\text{density}}\underbrace{\left(\frac{\pi D^2 L}{4}\right)}_{\text{volume}}\underbrace{g\sin\alpha}_{\text{gravity}} = -\rho\frac{\pi D^2}{4}g\Delta h$$

· add in body force to the x-force balance

$$-\frac{\Delta P \pi D^2}{4} - \rho g \Delta h \frac{\pi D^2}{4} - \tau_w \pi D L = 0$$

same dim

$$\tau_w = \frac{-D}{4}\frac{\Delta(P + \rho g h)}{L}\ ;\ \mathcal{P} = P + \rho g h$$

→ dynamic/modified pressure

$$\boxed{\tau_w = \frac{-D}{4}\frac{\Delta\mathcal{P}}{L}} = \frac{D}{4}\frac{|\Delta\mathcal{P}|}{L}$$

· Pipe Flow Equations

▫ general:

$$Re = \frac{Du\rho}{\mu} \quad f = \frac{\tau_w^2}{\rho u^2} \quad \tau_w = \frac{D}{4}\frac{|\Delta P|}{L}\ \text{(horizontal pipe)}$$

▫ laminar flow

$$f = \frac{16}{Re},\ Re \leq 2100$$

▫ turbulent flow: smooth pipes

Prandtl-Karman: $\frac{1}{\sqrt{f}} = 4\log(Re\sqrt{f}) - 0.4,\ Re \geq 3 \cdot 10^3$

Colebrook: $f = \left[3.6\log\left(\frac{Re}{6.9}\right)\right]^{-2},\ Re \geq 5 \cdot 10^3$

Blasius: $f = 0.0791 Re^{-\frac{1}{4}},\ 3 \cdot 10^3 \leq Re \leq 1 \cdot 10^5$

· Pipe Flow Summary

$f = \tau_w^2/\rho u^2 \quad Re = \frac{Du\rho}{\mu} \quad U = Q/A$

$$f = \frac{\tau_w^2}{\rho u^2} = \frac{D|\Delta\mathcal{P}|}{2L\rho u^2} = \text{function}(Re)$$

Roughness

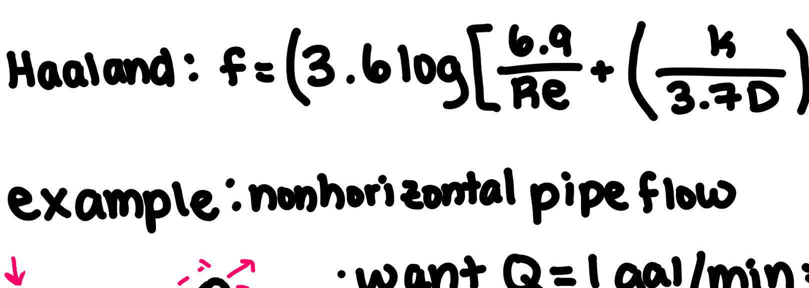
▫ table 2.2 in Deen

k = roughness

$[k] = L$

$f = fxn(Re, k/D)$

· Mean Velocity Profile



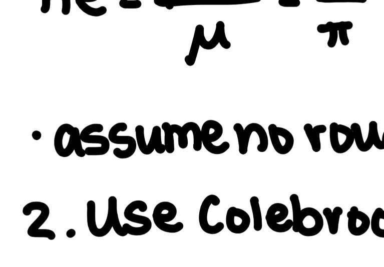
▫ Haaland Equation vs. Colebrook-White

↳ analogous to forms of smooth pipes above

↳ choose most solvable

Haaland: $f = \left(3.6\log\left[\frac{6.9}{Re} + \left(\frac{k}{3.7D}\right)^{1.11}\right]\right)^{-2}$

example: nonhorizontal pipe flow



· want $Q = 1\ \text{gal/min} = 6.3 \cdot 10^{-5}\ m^3/s$

· H_2O room temp, $\mu = 10^{-3}\ Pa \cdot s$, $\rho = 10^3\ kg/m^3$

· $D \ll L$ · assume steady

· what is $|\Delta P|$ needed?

1. Calculate Renold's Number

$$Re = \frac{\rho u D}{\mu} = \frac{4}{\pi}\frac{Q\rho}{D\mu} \approx 8034$$ turbulent!

· assume no roughness since no k given

2. Use Colebrook (could use Blasius!)

$$f = \left[3.6\log\left(\frac{Re}{6.9}\right)\right]^{-2} \approx 0.0082$$

3. Use f definition

$$f = \frac{D}{2\rho u^2}\frac{|\Delta\mathcal{P}|}{L}$$

$|\Delta\mathcal{P}| = 1.06 \cdot 10^4\ Pa$

$\Delta\mathcal{P} = -1.06 \cdot 10^4\ Pa$

$\Delta\mathcal{P} = \Delta P + \rho g \Delta h$

$$\boxed{\Delta P \approx -2 \cdot 10^4\ Pa}$$